

# Convex Optimization for Machine Learning

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Cs 460 - Machine learning  
Poster Presentation

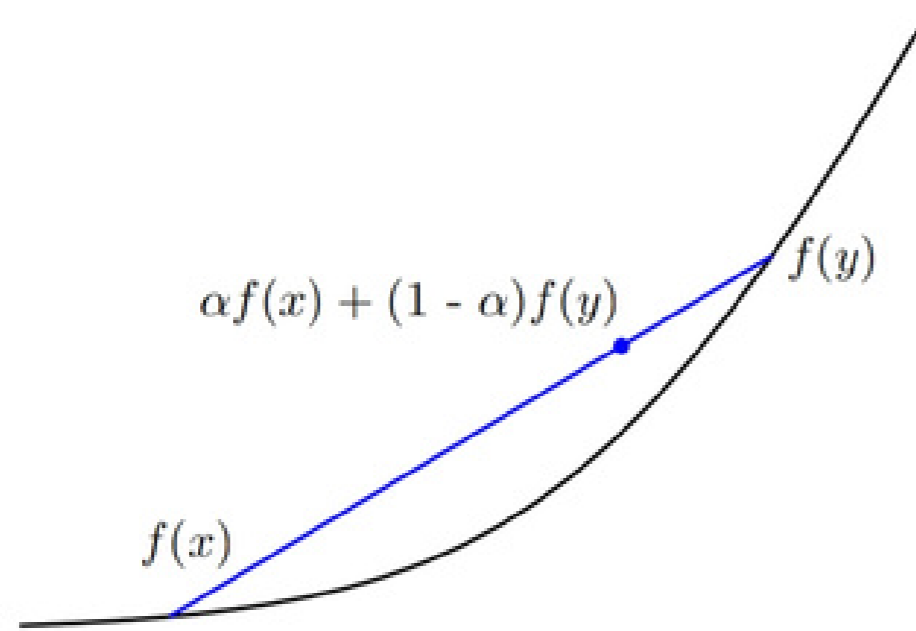
Guide :- Dr. Subhankar Mishra

## what is optimization ?

Finding the minimizer of a function subject to constraints :  
 minimize  $f(x)$   
 s.t.  $f_i(x) \leq 0, i = \{1,2,\dots,k\}$   
 $h_j(x) = 0, j = \{1,2,\dots,l\}$

## Convex Functions

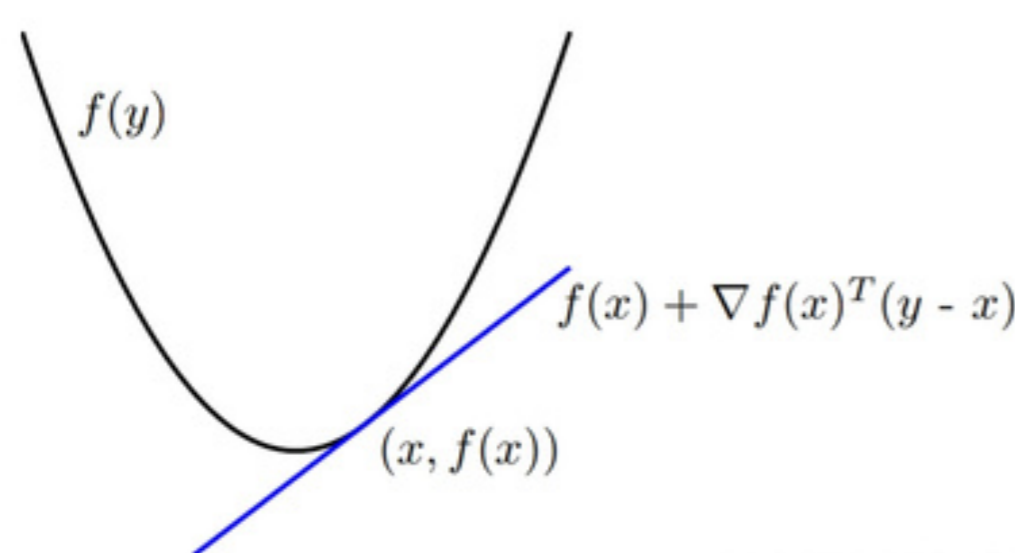
A function  $f: \mathbb{R}^n \rightarrow \mathbb{R}$  is convex if for  $x, y$  belonging to domain of  $f$  and any  $a$  belonging to  $[0,1]$ ,  
 $f(ax+(1-a)y) \leq af(x)+(1-a)f(y)$



## Convexity conditions

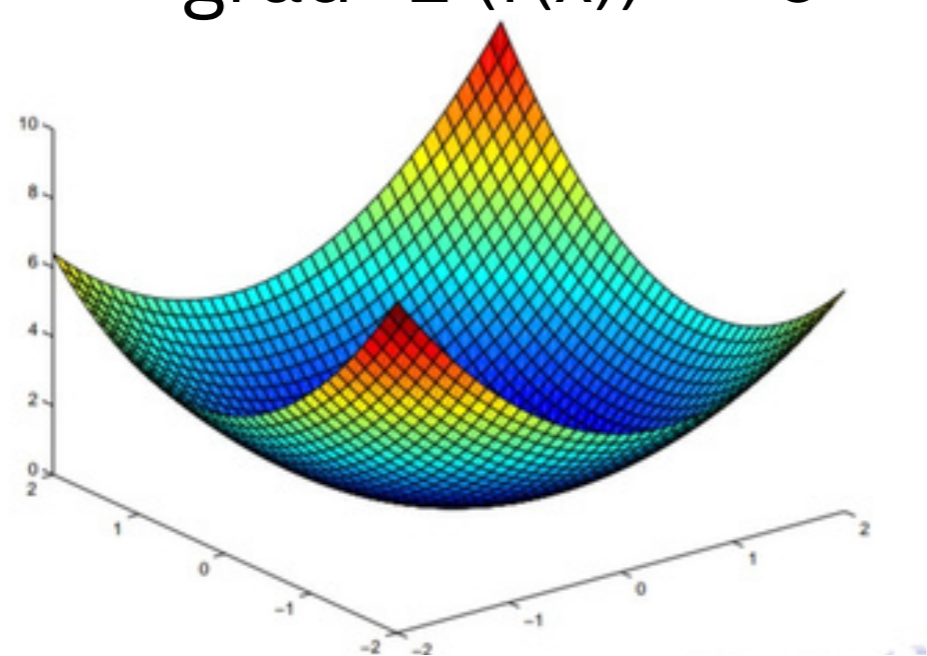
First order convexity condition

Theorem:- suppose  $f: \mathbb{R}^n \rightarrow \mathbb{R}$  is differentiable. Then  $f$  is convex iff for all  $x, y$  belonging to domain of  $f$ , we have  
 $f(y) \geq f(x) + \text{grad}(f(x))^T (y-x)$



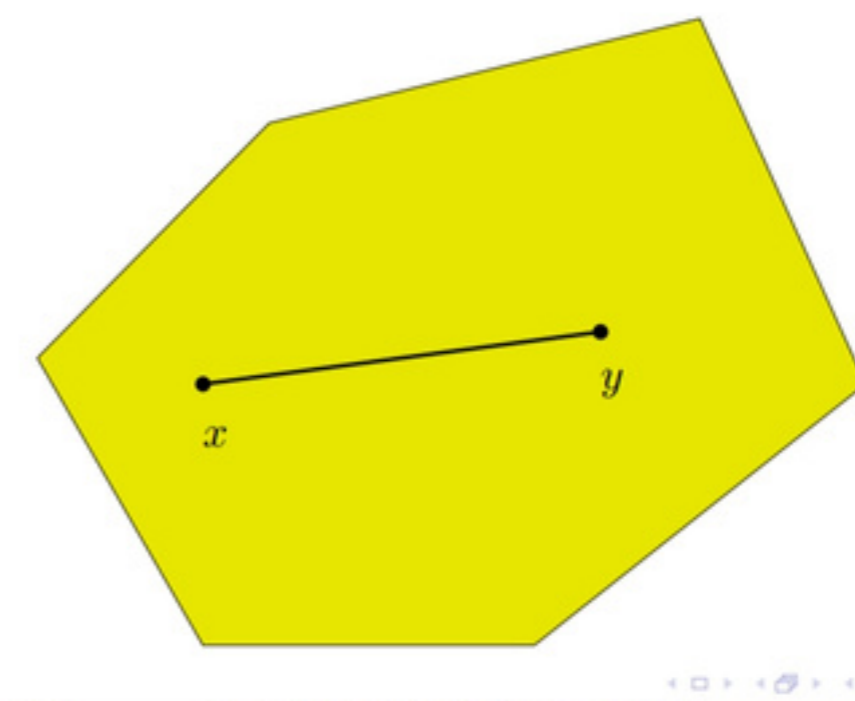
Second order convexity condition

Theorem:- suppose  $f: \mathbb{R}^n \rightarrow \mathbb{R}$  is twice differentiable. Then  $f$  is convex iff for all  $x$  belonging to domain  $f$  we have  
 $\text{grad}^2(f(x)) \geq 0$

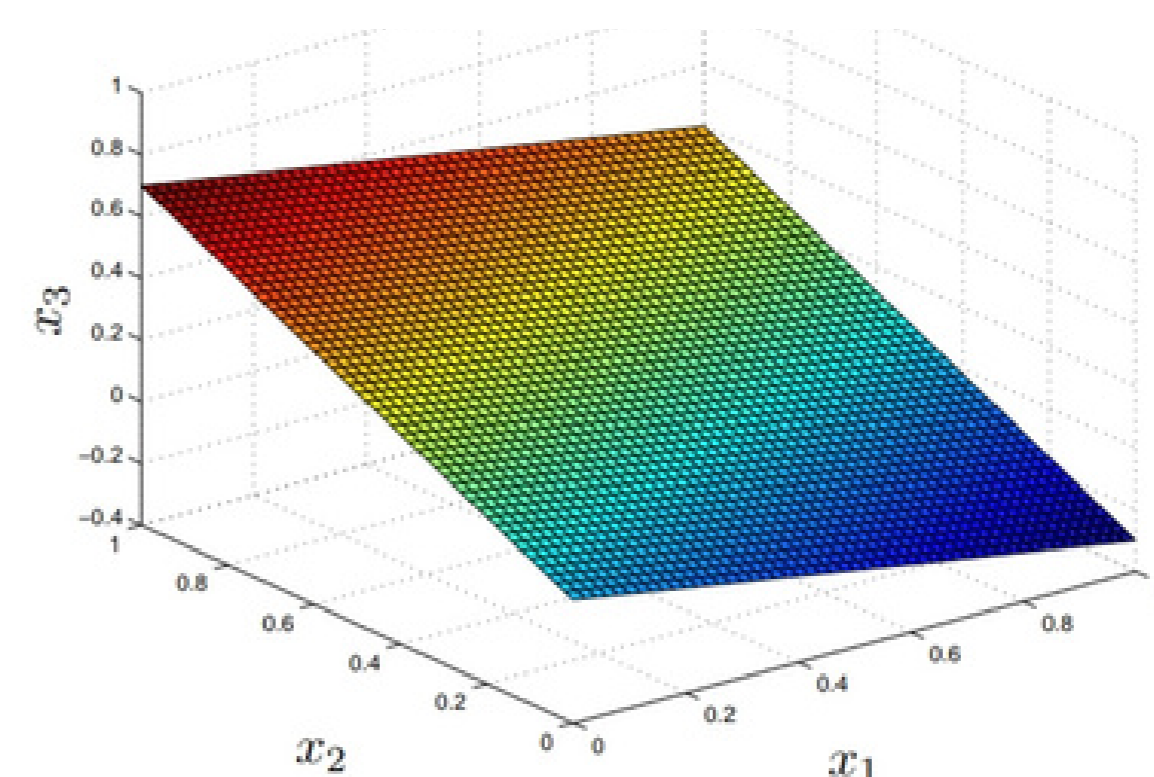


## Convex Sets

A set  $C$  is a subset of  $\mathbb{R}^n$  is convex if for  $x, y$  belonging to  $C$  and any  $a$  belonging to  $[0,1]$ ,  
 $ax + (1-a)y$  belongs to  $C$



Example :- Affine Subspaces :  $Ax = b, Ay = b$ , then  
 $A(ax+(1-a)y) = aAx+(1-a)Ay=ab+(1-a)b=b$



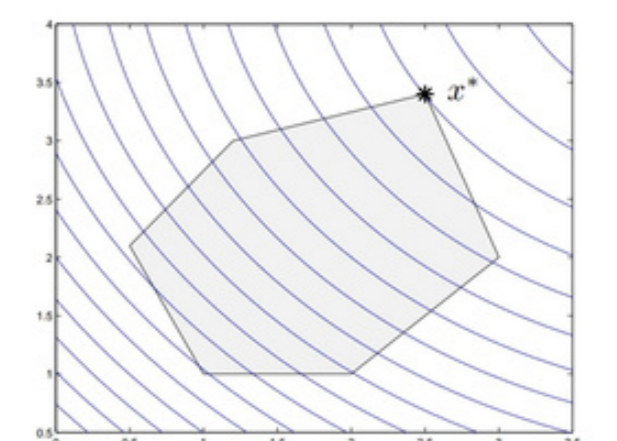
## Convex Optimization Problem

A optimization problem is convex if it's objective is a convex function, the inequality constraints  $f_i$  are convex, and the equality constraints  $h_j$  are affine

minimize  $f(x)$  (convex function)  
 s.t.  $f_i(x) \leq 0$ . (Convex sets)  
 $h_j = 0$ . (affine)

## Some important results

#. If  $x^*$  is a local minimizer of a convex Optimization Problem then it is a global minimizer



##. Gradient of  $f(x) = 0$  iff  $x$  is a global minimizer of  $f(x)$